Effect of Column Stiffness on Braced Frame Seismic Behavior

Gregory A. MacRae, M.ASCE1; Yoshihiro Kimura2; and Charles Roeder, M.ASCE3

Abstract: Steel concentrically braced frames are generally designed to resist lateral force by means of truss action. Design considerations for columns in these frames are therefore governed by the column axial force while column bending moment demands are generally ignored. However, if the columns cannot carry moments, then dynamic inelastic time-history analyses show that a soft-story mechanism is likely to occur causing large concentrated deformations in only one story. Such large concentrations of damage are not generally seen in real frames since columns are generally continuous and they possess some flexural stiffness and strength. This paper develops relationships for column stiffness and drift concentration within a frame based on pushover and dynamic analyses. It is shown that continuous seismic and gravity columns in a structure significantly decrease the possibility of large drift concentrations. An assessment method and example to determine the required column stiffness necessary to limit the concentration of story drift is provided.

DOI: 10.1061/(ASCE)0733-9445(2004)130:3(381)

CE Database subject headings: Bracing; Columns; Earthquake engineering; Framed structures; Seismic design; Seismic stability.

Introduction

Concentrically braced frames (CBFs) are not generally considered to perform as well in earthquakes as some other types of structure. This is reflected in the fact that response modification factors, R, for concentrically braced frames are generally less than those for eccentrically braced frames or moment-resisting frames (ICCI 2000). Two reasons for the lower performance expected for CBFs are the possibility of a pinched hysteresis loop, and the possibility of a soft-story mechanism.

A pinched hysteresis loop due to braces which buckle is often considered less desirable than a full hysteresis loop. Less earthquake energy is absorbed (Jain et al. 1978), concentration of strain occurs in the brace increasing its likelihood of fracture after a few cycles of loading, and impact forces occur in the structure as the braces straighten. However, these problems can be avoided in newer types of “unbonded” braces in which jackets prevent the braces from buckling allowing a full hysteresis loop (Clark et al. 2000).

Large local deformations may also occur at specific locations in braced frames, especially if the column flexural stiffness is low. This has been shown for a ten-story model of an eccentrically braced frame in which the columns moment resistance was neglected by providing pinned connections at the ends of column and brace members. Peak drifts were 5.26%. This is greater than the 2.63% obtained when the column moment resistance was considered (MacRae et al. 1990). This behavior may be understood for the two-story braced frame shown in Fig. 1. If the force distribution is triangular, then the base shear force, $V$, is related to the forces at levels 1 and 2, $F_1$ and $F_2$, according to Eqs. (2) and (3),

$$F_1 + F_2 = V \quad (1)$$

$$V_2 = F_2 = \frac{2}{3} V \quad (2)$$

$$V_1 = 3F_1 = V \quad (3)$$

If the stiffness of each story is the same, then $k = k_1 = k_2$, and the interstory displacements, $d_1$ and $d_2$, and the total displacements at each level, $\Delta_1$ and $\Delta_2$, are

$$d_2 = F_2 / k = \frac{2}{3} V / k \quad (4)$$

$$d_1 = (F_1 + F_2) / k = V / k = \Delta_1 \quad (5)$$

$$\Delta_2 = \Delta_1 + d_2 = \frac{2}{3} V / k + V / k = \frac{5}{3} V / k \quad (6)$$

The yielding braces are assumed elastic-perfectly plastic and the peak base shear force in the structure is the story yield force, $V_i$. The ratio of the displacement at the top, $\Delta_2$, to the top displacement at yield in the frame, $\Delta_2^y$, of the yielding structure is $\mu_i$ as shown in Eq. (7), and the displacement in the first story is given by Eq. (8). The first story ductility, $\mu_1$, is given in Eq. (9). The yield displacement of the first story, $\Delta_1^y$, is approximately equal to the product of the width of the braced bay, $B$, and the brace

---

1 Associate Professor, Dept. of Civil and Environmental Engineering, Univ. of Washington, Seattle, WA 98195-2700.
2 Associate Researcher, Architectural Engineering, Tokyo Inst. of Technology, 2-12-1 Meguro, Tokyo, Japan 152-8552.
3 Professor, Dept. of Civil and Environmental Engineering, Univ. of Washington, Seattle, WA 98195-2700.

Note. Associate Editor: Andrei M. Reinhorn. Discussion open until August 1, 2004. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on June 11, 2002; approved on March 28, 2003. This paper is part of the Journal of Structural Engineering, Vol. 130, No. 3, March 1, 2004. ©ASCE, ISSN 0733-9445/2004/3-381–391/$18.00.
material yield strain, $\epsilon_y$, as shown in Eq. (10), where $L_b=$brace length and $\theta=$angle of the brace from the horizontal,

$$\Delta_2=\mu_1 \Delta_2=\frac{5}{3} \mu_1 V_y / k$$

(7)

$$\Delta_1=\Delta_2-d_2=\frac{5}{3} \mu_1 V_y / k-\frac{2}{3} V_y / k$$

(8)

$$\mu_1=\Delta_1 / \Delta_{1y}=\frac{\frac{5}{3} \mu_1 V_y / k-\frac{2}{3} V_y / k}{(V_y / k)}=\frac{5}{3} \mu_1-\frac{2}{3}$$

(9)

$$\Delta_{1y}=L_b \epsilon_y \cos \theta=\beta / \cos \theta \epsilon_y \cos \theta=\beta \epsilon_y$$

(10)

Similarly a drift concentration factor ($DCF$), which describes the ratio of the maximum story drift, $\max (d_1, h_1, d_2, h_1)$, to the maximum roof drift, $\Delta_2 / H$, is given in Eq. (11), where $H=$structure height and $h_1$ and $h_2 =$heights of each story. This ratio is unity when the frame moves over linearly with height. Otherwise it is greater than unity.

$$DCF=\max (d_1, h_1, d_2, h_1) / (\Delta_2 / H) = 2-0.8 / \mu_1$$

(11)

It may be seen from Eq. (9) that if the total roof ductility, $\mu_1$, is 5, then the ductility demand in the first story, $\mu_1$, will be 7.66 and the $DCF$ from Eq. (11) is 1.84.

General equations for assessing the first story ductility demand, $\mu_1$, and $DCF$ are given in Eqs. (12)–(16) for frames with $N$ stories. It is assumed here that an arbitrary lateral force distribution is applied in one direction, that only the first story yields, and that the force-displacement behavior is elastic-perfectly plastic. Here $d_1$, $k_1$, $F_i$, $V_i$, and $h_j$ are the interstory drift for story $i$, lateral stiffness for story $i$, applied force at level $i$, story shear for level $i$, and the interstory height of level $i$, respectively. The first story displacement at first story yield is $d_{1y}=\Delta_{1y}$. The roof displacement at first story yield is $d_{1y}$ and the roof ductility is $\mu_1$. The $DCF$ is plotted for frames of different height subject to an inverted triangular load distribution in Fig. 2,

$$d_i=\frac{V_i}{k_i} \sum_{j=1}^{N} F_j$$

(12)

$$\Delta_{1y}=\sum_{i=1}^{N} d_i$$

(13)

$$\mu_1=\frac{\left(\mu_1 \Delta_{1y}-\sum_{j=2}^{N} d_i\right)}{\Delta_{1y}}=\frac{\left(\mu_1 \Delta_{1y}-\Delta_{1y}+\Delta_{1y}\right)}{\Delta_{1y}}=\frac{(\mu_1-1) \Delta_{1y}+1}{\mu_1}$$

(14)

$$\Delta_{1y}=\mu_1 \Delta_{1y}$$

(15)

$$DCF=\frac{\left(\mu_1-1\right) \frac{H}{h_1}}{\mu_1}$$

(16)

For tall frames subject to large ductility demands, the ratio of first story displacement to roof displacement at first story yield, $\Delta_{1y} / \Delta_{1y}$, is relatively small, so $DCF$ increases almost linearly with building height for a specific roof ductility, $\mu_1$, as shown in Fig. 2 and $DCF$ may be approximated using Eq. (17). This approximation is also reasonable for frames with irregular story heights and with other lateral force and stiffness distributions. For large ductilities, $\mu_1$, in a frame in which all stories have the same height, $h$, the $DCF$ approaches $H / h$ which is equal to the total number of stories, $N$. For a thirty-story frame with uniform story height and a roof ductility, $\mu_1$, of 5, $\Delta_{1y} / \Delta_{1y}=20.33$, $H / h=N/30$, so $\mu_1=82.3$ and $DCF=24.3$ according to Eq. (16) and $DCF=24$ according to Eq. (17). These large drift concentrations are best avoided in actual frames by using columns which are continuous over several stories,

$$DCF=\frac{\left(\mu_1-1\right) \frac{H}{h_1}}{\mu_1}$$

(17)

While researchers have been concerned about the possibility of drift concentration in frames (e.g., Paulay 1978; Akiyama and Ohi 1981; Uetani and Tagawa 1998; Krawinkler and Gupta 1998), no study has been performed to quantify the column stiffness and strength necessary to decrease excessive drift concentration. Such a study has not been performed possibly because many typical frames tend to perform satisfactorily without explicitly studying drift concentration. However, understanding the relationship between continuous column properties and frame performance means that the designer does not need to simply hope that the structure will behave satisfactorily, but the designer can “tell the structure what to do.”

The study described in this paper (i) develops closed form relationships between drift concentration, column stiffness, and
Behavior of a Two-Story Frame with Continuous Columns

Continuous columns over the height of the frame, whether they are gravity or seismic columns, resist the tendency for concentration of deformation in one story. The amount of drift concentration will depend on the column flexural stiffness. If the columns are flexurally rigid, then the frame DCF is unity. Fig. 3 shows a two-story frame with the same strength and stiffness at each level. One “continuous column,” representing the flexural stiffness of all continuous columns in the frame, is shown. The column is assumed to be pinned at its base. The pushover behavior of this two-story frame is described in the following (Kimura et al. 2002).

The shear force in the gravity column, \(V_c\), is the same in the first story, \(V_{c1}\), and the second story, \(V_{c2}\), and is related to the force applied at the center of the column, \(P\), as well as to the flexural stiffness, \(EI\), interstory height, \(h\), and to displacement at the center of the column, \(d_c\),

\[
V_{c1} = V_{c2} = V_c = P/2 = 48EI/d_c \times (h/2)^3/3 = 3EI/d_c \times h^3
\]  
(18)

Since \(d_c = (\Delta_1 - \Delta_2)/2\), then

\[
V_c = 3EI(\Delta_1 - \Delta_2)/2h^3
\]  
(19)

The frame shears before yield are

\[
V_{f2} = V_2 - V_c = \Delta_1 (3EI/2h^3) = k(\Delta_2 - \Delta_1)
\]  
(20)

\[
V_{f1} = V_1 + V_c = V - (3EI/2h^3)k\Delta_1
\]  
(21)

Solving for \(\Delta_1\) and \(\Delta_2\) from Eqs. (19) and (20):

\[
\Delta_1 = \frac{V(2 + 5\alpha)}{k(2 + 6\alpha)} = d_1
\]  
(22)

where \(\alpha\) is the column stiffness ratio:

\[
\alpha = \frac{EI}{kh^3}
\]  
(25)

The DCF is obtained from Eq. (10) and \(d_2\) is obtained using Eq. (8) as shown in Eq. (26),

\[
DCF = \frac{2\Delta_1}{\Delta_2} = \frac{6(2 + 5\alpha)}{5(2 + 6\alpha)}
\]  
(26)

The column moment, \(M_c\), is

\[
M_c = V_c h = \frac{\alpha}{(2 + 5\alpha)} V_{f1}h
\]  
(27)

The story yield force considering the presence of the continuous column, \(V_{f1y}\), divided by that ignoring the column, \(V_{f1}\), for the two-story frame is given in the following. The maximum increase is 20%.

\[
V_{f1y} = V_{f1} \frac{2 + 6\alpha}{2 + 5\alpha}
\]  
(28)

It was shown that the fundamental period of the frame decreased by less than 3% due to the presence of the continuous column for frames of various heights (Kimura et al. 2002). This change in period is probably small enough to be ignored.

When the frame becomes inelastic, and the bottom story yields the shear force in the bottom story of the frame, \(V_f1\), becomes equal to the yield shear force in that story \(V_{f1y}\). Relationships for the drift concentration factor, DCF, and the column moment, \(M_c\), are obtained for different column stiffnesses, \(\alpha\), and for different ductilities, \(\mu_s\), in the following.

\[
DCF = \frac{10\mu_s + 55\mu_s \alpha - 4 - 10\alpha + 75\alpha^2 \mu_s}{5\mu_s(3\alpha + 1)(1 + 3\alpha)}
\]  
(29)

\[
M_c = \frac{(15\alpha \mu_s + 5\mu_s - 4 - 10\alpha)}{(2 + 5\alpha)} V_{f1}h
\]  
(30)

When both the top and bottom stories yield, \(V_{f1} = V_{f2} = V_f\) and the expressions become:

\[
DCF = \frac{2d_1}{\Delta_2} = 1 - \frac{1}{15\mu_s \alpha}
\]  
(31)

\[
M_c = V_c h = 0.2V_f h = 0.1V_{f1}h
\]  
(32)

Fig. 4(a) compares actual and theoretical relationships for DCF for specified ductility levels. Analysis A shows the DCF from Eqs. (29) and (31). The two curves shown depend on which equation governs. Analysis B shows the DCF from a frame analysis using the computer program DRAIN-2DX (Prakash et al. 1993). Here, the beam and column members were made rigid axially, and the frame members were pinned at their ends. Analyses A and B are consistent. As would be expected, the DCF tends to unity as the column flexural stiffness increases. It may be seen from Fig. 4(a) that the column is effective in reducing story drift concentration when the column stiffness ratio, \(\alpha\), is greater than about 0.1.
Fig. 4 compares actual and theoretical relationships for maximum column moment demand for specified ductility levels from Eqs. (29) and (31). For columns with high stiffness and large ductilities, the maximum column moment occurs at second-story yield. For more flexible columns subject to lower ductility demands, the top story remains elastic at the target ductility. For values of $\alpha$ greater than 0.1, the column moment is at its maximum value for large ductilities so columns should be designed for this maximum moment according to Eq. (32).

Dynamic inelastic time history analyses were carried out (Kimura et al. 2002) and indicated that the static DCF was not exceeded but there was significant variation in the column moment.

Multistory Frame Demands from Inverse Triangular Force Distribution

For a continuous structure subject to an inverse triangular load distribution, $w = w_o x / H$, where $x$ is the distance up the frame from the base and the system base shear, $V_{s, base} = w_o H / 2$. The shear, $V$, and moment, $M$, up the height of the frame are given in the following as illustrated in Fig. 5(a):

$$V = V_{s, base} = \int w \, dx = \frac{w_o H}{2} \int \frac{w_o x}{H} \, dx = \frac{w_o H}{2} \left(1 - \left(\frac{x}{H}\right)^2\right)$$

$$M = \int V \, dx = \frac{w_o H^2}{6} \left(2 + \left(\frac{x}{H}\right)^3 - 3 \left(\frac{x}{H}\right)\right)$$

If there is no continuous column then the uniformly distributed load up the height is $w' = w_o x / H$ and $w_o'$ is the distributed force at the top of the frame which is related to the frame base shear strength, $V_{f, base}$, according to Eq. (35) and the shear force distribution if there is no column, $V_{fned}$, is given in Eq. (36),

$$w_o' = 2 V_{f, base} / H$$

$$V_{fned} = V_{f, base} \left(1 - \left(\frac{x}{H}\right)^2\right)$$

Fig. 5. System, frame, and strong column behavior due to pushover analysis at large displacements
Multistory Frame Roof Displacement with No Continuous Column

Most tall braced frames do not have constant strength over their heights but the strength decreases with height as the shear demand decreases. One way to describe the distribution of strength up the height is with the parameter, $\beta$. The frame shear strength capacity, $V_{fc}$, is given in Eq. (37) using Eq. (36). When $\beta=1$, $V_{fc}$ is always equal to the capacity of the frame at the base $V_{f,base}$ and the frame has uniform strength over its height. If $\beta=0$, then $V_{fc}$ is equal to the shear strength demand assuming no continuous column, $V_{fncd}$, and all stories will yield simultaneously without drift concentration under this inverse triangular force regime even if there is a continuous column as shown in Fig. 5(b). Since the shear strength distribution is the same as the shear stiffness distribution, $GA$, in a CBF, $GA$, is given in Eq. (38) where $GA_{base}$ is equal to the shear stiffness at the base of the frame. The elastic drift, $\theta$, is equal to the shear demand, $V_{f}$, from Eq. (36) divided by $GA$ as shown in Eq. (39). The displacement of the structure may be found by integrating Eq. (39) to obtain Eq. (40). This expression is reasonably complex. An approximation to the elastic peak displacement at the top of the frame if there is no continuous column, $\Delta_{ynct}$, is given in Eq. (41), where the stiffness of a discrete story, $k$, may be taken as $GA/h$. Here linear interpolation was used between the cases when $\beta=0$ and $\beta=1$ according to Eq. (41). The elastic $DCF$ at first yield is $3/(3-\beta)$,

$$V_{fc} = V_{fncd} + \beta (V_{f,base} - V_{fncd})$$

$$= V_{f,base} \left(1 - \frac{x^2}{H^2}\right) + \beta \left(V_{f,base} - V_{f,base} \left(1 - \frac{x^2}{H^2}\right)\right)$$

$$= V_{f,base} \left(1 - (1-\beta) \left(\frac{x}{H}\right)^2\right)$$

$$GA = G_A_{base} \left(1 - (1-\beta) \left(\frac{x}{H}\right)^2\right)$$

$$\theta = \frac{V}{GA} = \frac{V_{f,base} \left(1 - \frac{x^2}{H^2}\right)}{G_A_{base} \left(1 - (1-\beta) \left(\frac{x}{H}\right)^2\right)}$$

$$\Delta = \frac{V_{f,base} H}{G_A_{base} (1-\beta)} \left(\frac{x}{H} \cdot \frac{\beta}{2 \sqrt{1-\beta}} \ln \left(1 + \sqrt{1-\beta} \frac{x}{H}\right)\right)$$

$$\Delta_{ynct} = \frac{(3-\beta) V_{f,base} H}{3 G_A_{base}} = \frac{(3-\beta) V_{f,base} N h}{3 G_A_{base}} = \frac{(3-\beta) V_{f,base} N}{3 k}$$

$$= \Delta_{1y} N \left(1 - \frac{\beta}{3}\right)$$

The top displacement, $\Delta_{1y}$, is equal to the yield roof displacement assuming no continuous column, $\Delta_{ynct}$, multiplied by the ductility factor, $\mu$, as shown in Eq. (42). The $DCF$ if there is no column is given by Eq. (16) as described previously.

$$\Delta = \mu \Delta_{ynct} = \mu (1 - \beta/3) N \Delta_{1y}$$

Continuous Frame/Continuous Column Pushover Behavior After Frame Mechanism Formation

When continuous columns exist with a continuous frame in which displacements are sufficient to cause all levels of the frame to yield, then column shear demands may be found from the frame-column system shear demand and the frame shear capacity. This difference is represented by the parameter $\beta$ and it is independent of the column stiffness ratio, $\alpha$. The frame distributed forces, $w_{cf}$, are given by Eq. (43) and the point force at the top of the frame, $F_{f,roof}$, must be equal to the shear strength at the top of the frame as shown in Fig. 5,

$$w_f = \frac{dV_f}{dx} = \frac{2x}{H^2} V_{f,base} (1-\beta) = w_o (1-\beta) = w_{f,ln}$$

$$F_{f,roof} = \beta V_{f,base}$$

The continuous column top point force, $F_{c}$, and distributed forces, $w_{c}$, are equal to the system forces minus the frame forces. Therefore:

$$F_{c} = -\beta V_{f,base}$$

$$w_{c} = w_f - w_{f,ln} = \frac{w_o x}{H} (1-\beta) = \frac{x}{H} (w_o - w_{o,ln} + \beta w_{o,ln}) = \frac{x}{H} w_{co}$$

Since the moment at the base of the continuous column, $M_{cc,base}$, is zero as shown in Eq. (46), the distributed force at the top of the column, $w_{co}$, and the total distributed force at the top of the system, $w_{o,ln}$, may be found according to Eqs. (47) and (48). If $\beta=1.0$, then the distributed force resisted by the system, $w_{o,ln}$, is 1.5 times that if there is no column, $w_{o,ln}$. The total system base shear resistance, $V_{f,base}$, is therefore 1.5 times that of the frame with no column, $V_{f,ncd}$.

$$M_{cc,base} = 0 = F_c H + w_{co} H 2H \frac{H}{3} = -\beta V_{f,base} H + w_{co} H \frac{H}{3}$$

$$w_{co} = 3 \beta V_{f,base} H / H = 3 \beta w_{o,ln} / 2$$

$$w_{o} = w_{co} + w_{o,ln} - \beta w_{o,ln} = w_{o,ln} (3 \beta / 2 + 1 - \beta) = w_{o,ln} (1 + 0.5 \beta)$$

The shear force at the base of the column, $V_{f,base}$, is given in Eq. (49) and up the height of the frame in Eq. (50). The maximum moment, $M_{c,max}$, will occur where the shear force, $V_{c}$, is zero. That is, when $x = H / \beta = 0.5773H$. The continuous column moment distribution, $M_{c,e}$, and the maximum column moment at mechanism formation, $M_{c,mech}$, at $x = 0.5773H$, are given in Eqs. (51) and (52).

$$V_{f,base} = F_c + 3 \beta w_{o,ln}/2 H / 2 = -\beta V_{f,base} + 3 \beta V_{f,base} / 2$$

$$= \beta V_{f,base} / 2$$

$$V_{c} = \beta \frac{V_{f,base}}{2} \left(1 - 3 \left(\frac{x}{H}\right)^2\right)$$

$$M_{c,e} = \beta \frac{V_{f,base} x}{2} \left(1 - \left(\frac{x}{H}\right)^2\right)$$

$$M_{c,mech} = \beta \frac{V_{f,base} H}{3 \sqrt{3}} = 0.192 \beta V_{f,base} H$$
Discrete Frame/Continuous Column Pushover Behavior After Frame Mechanism Formation

The actual moment for frames is different from that assumed for a continuous column, since the loading is discrete, rather than continuous. As the number of floors increases, the continuous equation gives a better fit to the actual results. Eqs. (53) and (54) were empirically developed from frame pushover analyses using DRAIN-2DX (Prakash et al. 1993) to give a realistic estimate of the frame moment demand if there are a discrete number of levels, $M_{c, mech}$, assuming the frame forms a mechanism. It may be seen in Fig. 6 that this equation is appropriate for frames with five or more stories,

$$M_{c, mech} = 0.192V_f \cdot \text{base} \cdot H \beta'$$

$$\beta' = \left(1 - \frac{1.3}{N}\right)^{0.25 + (0.5/N)}$$

Rotations, $\theta$, at the top and bottom of a pin-ended column which is subjected to linearly increasing distributed loading form 0.0 to $w_{c, \gamma}$ at the top, as shown in Fig. 5(c), are 0.022 $22w_{c, \gamma}H^3/EI$ and 0.019 $44w_{c, \gamma}H^3/EI$, respectively, and the maximum displacement, $\Delta$, is 0.006 $53wH^3/EI$ at 0.5195 $H$ (Young and Budynas 2001). The rotation of the base of the continuous column relative to a line between the column ends, $\theta_{cb, m}$, is therefore given in Eq. (55). The peak frame drift once a mechanism has been formed, $\theta_{1m}$, is given in Eq. (56), and the $DCF$ is given in Eq. (57), where $H = Nh$, and $\Delta, \mu = \mu_\text{uneq}$, $\mu$ is the ductility relative to the displacement with no column, and $N$ is the number of stories in the frame,

$$\theta_{cb} = 0.01944 \frac{w_{c, \gamma}H^3}{EI} = 0.01944 \frac{3\beta w_{c, \gamma}H^3}{2EI} = 0.0583 \frac{\beta V_f \text{base} H^2}{EI}$$

$$= 0.0583 \frac{\beta k \Delta_{cb} (Nh)^3}{EH} = 0.0583 \frac{\beta \Delta_{1m} (Nh)^3}{\alpha H}$$

$$\theta_{1m} = \frac{\Delta_{1m}}{H} + \theta_{cb, m} = \frac{\Delta_{1m}}{H} + 0.0583 \frac{\beta \Delta_{1m} N^3}{\alpha H}$$

$$DCF = \frac{\theta_{1m}}{(\Delta_i / H)} = 1 + 0.0583 \frac{\beta \Delta_{1m} N^3}{\alpha \Delta_i} = 1 + 0.0583 \frac{\beta \Delta_{1m} N^3}{\alpha \mu \Delta_{uneq}}$$

$$= 1 + 0.0583 \frac{\beta \Delta_{1m} N^3}{\alpha \mu (1 - \beta/3)}$$

$$= 1 + 0.0583 \frac{\beta N^2}{\alpha (1 - \beta/3)}$$

(57)

Discrete Frame/Continuous Column Pushover Behavior at Frame Mechanism Formation

The above-developed expressions are only valid after a full mechanism has occurred. A mechanism will occur in a system loaded with the inverse triangular lateral force distribution when the drift at the top of the frame, $\theta_{1m}$, is equal to the top story yield displacement, $d_{1y}$, divided by the top story height, $h$, according to Eq. (58). The total rotation at the top of the frame, $\theta_{1m}$, will therefore also be equal to the roof displacement when a mechanism forms, $\Delta_{1m}$, divided by the structure height, $H$, minus $\theta_{1m}$ as shown in Eq. (58). The continuous column is subject to the inverse triangular loading, $w_c$, shown in Fig. 5(c), so the pin-ended column top rotation, $\theta_{cm}$, is $w_c H^3/4EI \beta \Delta_{1m} N^3/15\alpha H$. The top displacement, $\Delta_{1m}$, is therefore given in Eq. (59),

$$\theta_{1m} = d_{1y}/h = \Delta_{1m}/h = B e_y/h = \Delta_{1m}/H - \theta_{cm}$$

$$\Delta_{1m} = H(\theta_{1m} + \theta_{cm}) = H\left(\frac{\Delta_{1y}}{h} + \frac{\beta \Delta_{1m} N^3}{15\alpha H}\right)$$

$$= \Delta_{1y}N \left(1 + \frac{15\alpha}{\alpha N}\right)$$

(58)

At the point the mechanism occurs, the first-story drift and the frame $DCF$ are found by substituting Eq. (59) into Eq. (56):

$$\theta_{1m} = \frac{\Delta_{1y}N}{H} \left(1 + \frac{\alpha N^2}{8\alpha}ight)$$

$$= \frac{\Delta_{1y}N}{H} \left(1 + \frac{\beta N^2}{8\alpha}ight)$$

(59)

The $DCF$ at formation of a mechanism, $DCF_{1m}$, is therefore equal to

$$DCF_{1m} = \frac{\theta_{1m}}{\Delta_{1m}/H} = \frac{\Delta_{1y}}{H} \left(1 + \frac{\beta N^2}{8\alpha}\right)$$

$$= \frac{\Delta_{1y}}{H} \left(1 + \frac{\beta N^2}{15\alpha}\right)$$

(60)

Discrete Frame/Column Pushover Behavior Prior to Frame Mechanism

When the continuous column stiffness ratio, $\alpha$, is low, then the frame may not form a full yield mechanism at the expected maximum roof displacement demand, $\Delta_i$. While a mathematical closed form estimate of the column moment to estimate the likely behavior could possibly be developed, such a formulation is complex. Therefore Eq. (62) was developed empirically such that the column moment demand, $M_{c, max}$, increases with increasing roof displacement, $\Delta_i$, until the mechanism displacement, $\Delta_{1m}$, is ob-
obtained, and the factor of 0.65 was used to fit the results of pushover analyses. The term \( \Delta_s/\Delta_{tm} \) is obtained using Eqs. (42) and (59). The relationship of column moment, \( M_{c,max} \), to column stiffness, \( \alpha \), is given in Fig. 7(a). This response is similar to that obtained from pushover analyses,

\[
M_{c,max} = M_{c,mech} \left( \frac{\Delta_s}{\Delta_{tm}} \right)^{0.65} < M_{c,mech}
\]

\[
= M_{c,mech} \left( \frac{\mu(3-\beta)}{3+\beta N^2/5\alpha} \right)^{0.65} < M_{c,mech} \tag{62}
\]

If it were assumed that the deformed shape of the continuous column is the same as that when a full mechanism occurs, and that the magnitude of the shape increases linearly with increasing top displacement until the top-story mechanism displacement is reached, then the column base rotation within the pin-ended column, \( \theta_{cb} \), is given in Eq. (64) and the DCF is given in Eq. (65).

It may be seen that this expression for DCF is identical to that obtained in Eq. (61) and is independent of ductility,

\[
\theta_{cb} = \theta_{cbm}(\Delta_s/\Delta_{tm}) \tag{64}
\]

\[
DCF_m = \frac{\theta_{cbm}(\Delta_s/\Delta_{tm})}{\Delta_s/H} = \frac{H\theta_{cbm}}{\Delta_{tm}} = 1 + \frac{7\beta N^2}{120(\alpha + \beta N^2/15)} \tag{65}
\]

This equation indicates that the DCF before a full mechanism forms is independent of ductility. This is consistent with actual plots where the ductility is large. However, the maximum value of DCF according to Eq. (65) is 1.87 when \( \alpha = 0 \). This value is much less than that of Eq. (16), indicating that Eq. (65) may severely underestimate DCF. The reason for the difference for low values of \( \alpha \) is that frame yield may actually only affect a few stories near the base of the frame, not all stories over the height as anticipated in the above-mentioned approach. An equation to deal with this behavior is described in the following.

Eq. (66) was developed to describe DCF when the maximum moment in the continuous column occurred below the column mid-height. Here \( f_n(N) \) indicates a function of the number of stories, \( N \). The height over which a frame mechanism formed was obtained using a plastic analysis approach similar to that of Pekcan et al. (1997). This height of the mechanism was low for low values of \( \alpha \). It was assumed that as the roof drift increased the deformation at the mechanism height also increased. Eq. (67a) is an empirical equation in a similar form to Eq. (66) to better fit the pushover analysis results for ductilities, \( \mu \), greater than or equal to 2 and it is appropriate when the peak continuous column moment occurs below the column mid-height. The value of DCF from Eq. (65) is appropriate for stiffer columns in which a full frame mechanism is given in Eq. (67b). Pushover analysis results are similar to those from Eqs. (67a) and (67b) as shown in Fig. 7(b).

\[
DCF_a = \frac{1 + (\mu - 1)(1 - \beta/3) \left( \frac{\beta}{\alpha} \right)^{0.319}}{\mu(1 - \beta/3)} \frac{fn(N)}{\beta^{0.319}(\alpha^{0.223})} N^{0.35} \tag{66}
\]

\[
= \max \left\{ 1 + \frac{7\beta N^2}{120(\alpha + \beta N^2/15)} \right\} \tag{67a}
\]

\[
DCF_b = \frac{1 + (\mu - 1)(1 - \beta/3) \left( \frac{\beta^{0.319}}{\alpha^{0.223}} \right) N^{0.35}}{\mu(1 - \beta/3)} \tag{67b}
\]

Effect of Column Axial Stiffness on Response

In the modeling described previously, frames analyzed acted as shear structures since the beams and columns were assumed to have high axial stiffnesses. In order to investigate the effect of more realistic column axial stiffness, a twenty-story frame was analyzed with both very stiff and realistic column axial stiffnesses. For the twenty-story frame with no gravity column and \( \beta = 0.321 \), the period increased by 14\% from 1.40 to 1.59 s when column axial shortening was modeled and maximum axial force expected in the column at the initiation of a frame mechanism was 0.4 of the section yield force. Even though the frame was significantly more flexible than before, the DCF changed by less than 3\% and maximum column moments were always conserva-
tively estimated when the frame was pushed to roof ductilities of 1, 2, 4, and 6. This indicates that for the purposes of computing these two quantities, the effect of column axial deformation can be ignored.

Dynamic Effects on Frames

During earthquake shaking the DCF and column moment may be significantly different than that from inelastic pushover analysis since dynamic effects cause the distribution of inertial forces to change continually during the analysis. To quantify these differences concentrically braced frames were designed for Los Angeles seismic conditions according to the IBC assuming soil condition $S_D$ and assuming the inverted triangular static lateral force distribution. Three methods were used to design the frames. Case 1 frames were designed considering that the brace size, and hence story strength and stiffness, was constant over the height of the frame so $\beta = 1$. Case 2 frames were designed considering that the brace sizes changed every two stories. The brace strength in the lowest two stories exactly matched the demand at the base of a frame with no continuous columns. The brace strength for every two stories further up the frame was determined as the average of the shear force demand at the lower of the two levels considered and the shear force capacity from the level below. The value $\beta$ was found by multiplying both sides of Eq. (37) by the interstory height, $h = dx$, and carrying out an integration to obtain Eq. (68). Case 3 frames were designed so that the brace sizes changed every two stories. The brace strength in the lowest two stories exactly matched the demand at the base of a frame with no continuous columns. The brace strength for every two stories further up the frame was determined as the average of the shear force demand at the lower of the two levels considered and the shear force capacity from the level below. The value $\beta$ was found by multiplying both sides of Eq. (37) by the interstory height, $h = dx$, and carrying out an integration to obtain Eq. (68). Case 3 frames were designed so that the brace sizes changed every two stories. The brace strength in the lowest two stories exactly matched the shear force demand in each story. Beam and column members and connections were assumed to be strong enough such that "inelastic action will occur primarily through tension yielding and/or buckling of braces" (AISC 1997),

$$\beta = \frac{3 \sum_{i=1}^{N} V_i}{NV_{f,\text{base}}} \cdot 2$$  \hspace{1cm} (68)

Only the continuous column possessed moment capacity at the nodes, all other connections were pinned. Braces were considered to be unbonded with identical yield characteristics in tension and compression and with an elastic perfectly plastic hysteresis loop. Damping was applied as 2% of critical in the first and Nth mode using a Rayleigh damping model where $N$ is the number of stories. Newmark’s constant average acceleration integration method was used in the inelastic dynamic time history analysis. The first mode response periods for the two-, five-, ten-, and twenty-story frames when $\alpha = 0.0$ were 0.328, 0.538, 0.815, and 1.297 s, respectively. For Case 1 frames with $\beta = 1$ and they were 0.351, 0.616, 0.927, and 1.549 s. For Case 3 frames with $\beta$ close to 0.0. For the 2, 5, 10 and 20 story Case 2 frames $\beta = 0.750, 0.650, 0.506$ and 0.301 respectively and the periods were 0.337, 0.557, 0.854, and 1.398 s.

These frames were analyzed with six records, La01, La03, La05, La21, La23, and La25 obtained from the SAC database (Somerville et al. 1997). Approximate probabilities of exceedance for the first three earthquake records listed above were 10% in 50 years and they were 2% in 50 years for the fourth, fifth, and sixth records.

The dynamic drift concentration factor, $DCF_d$, was computed from the peak story drift and the peak roof drift even though these can occur at different times. For all analyses when there was no column ($\alpha = 0$), $DCF_d$ was similar to the DCF from a static analysis. Also, in the case with a rigid continuous column ($\alpha = \infty$), $DCF_d$ was always equal to unity as would be expected. Fig. 8a shows the dynamic strength reduction factor, $DCF_d$, for frames with different $\beta$ plotted against the static value for uniform strength distribution ($\beta = 1$), $DCF_{\beta=1}$. It may be seen that $DCF_d$ is generally less than or equal to $DCF_{\beta=1}$. The $DCF_d$ may become greater than $DCF_{\beta=1}$ for a structure with $\beta = 1$ as a result of the cyclic loading effects and the changing lateral force distribution. While $DCF_{\beta=1}$ gives a good upper-bound indication of the $DCF_d$ it may be too high for design. Fig. 8b, which is based on Eq. (69), gives a more realistic average estimate. This is based

![Fig. 8. Relationship between dynamic drift concentration factor, $DCF_d$, and static $DFCs$ for all frames](image-url)
on the observation that for Case 3 structures, which have a static $DCF = 0$, the dynamic factor, $DCF_d$, was similar to those for frames with $\beta = 0.3$, while for frames with higher values of $\beta$, the static and dynamic concentration factors, $DCF$, and $DCF_d$ were not dissimilar,

$$DCF_d = \begin{cases} 
DCF_{\beta=0.3} & \beta < 0.3 \\
DCF & \beta \geq 0.3 
\end{cases} \quad (69)$$

The maximum column moment will also change due to dynamic effects which changes the lateral force distribution. If the lateral force distribution is uniformly distributed then the peak column moments may be obtained using the approach described previously for a triangular load distribution. These moments are an additional $0.05V_{f,base}H$, $0.051V_{f,base}H$, and $0.058V_{f,base}H$ for $\beta$ values of 0, 0.5, and 1.0, respectively. The coefficient in front of $V_{f,base}$ differs by less than 20% in these cases indicating that the dynamic moment, caused by different load distributions, may be equal to the static moment plus some additional term. This moment should also be zero in structures with no columns ($\alpha = 0$). Equation (70) was developed empirically to fit the dynamic column moment, $M_{c,max,d}$, as a function of the static moment, $M_{c,max}$. Fig. 9 shows that there is still considerable scatter.

$$M_{c,max,d} = M_{c,max} + 0.0054(0.02)V_{f,base}H\mu^{0.875} \alpha^{0.36} N^{1.25}$$

$$= M_{c,max} + 0.000108V_{f,base}H\mu^{0.875} \alpha^{0.36} N^{1.25} \quad (70)$$

Frame Story Demand Assessment Procedure

Steps to estimate maximum story drift and continuous column moment demand are given in the following.

1. Estimate peak roof displacement of structure, $\Delta_r$, by standard procedures such as the equal displacement method or a rational code approach. From elastic analysis find the roof displacement corresponding to yield of the braces, $\Delta_y$.

2. Compute key parameters for the frame. The roof ductility demand, $\mu$, may be found as $\Delta_r/\Delta_y$. The frame base shear strength, $V_{f,base}$, may be computed from the base story brace strengths. The parameter $\alpha$ may be found from Eq. (25) where $k$ is the lateral stiffness of the base story ignoring column axial stiffness, and $\beta$ may be estimated from Eq. (68).

3. Compute $\beta'$, $M_{c,mech}$, $M_{c,max}$, and $DCF$ using Eqs. (54), (53), (63), and (67), respectively.

4. The maximum likely drift concentration, $DCF_d$, and column moment, $M_{c,max,d}$, considering dynamic effects may be estimated using Eqs. (69) and (70), respectively.

5. The maximum story drift is given as

$$\theta_{max} = \left( \frac{\Delta_r}{H} \right) DCF_d$$

6. If $\theta_{max}$ is greater than the acceptable drift limit or if the moment demand on the continuous columns, $M_{c,max,d}$, is greater than the column strength then the size of the gravity columns should be increased or the frame should be strengthened.

It should be noted that the above-noted procedure has a number of limitations. It is assumed that the continuous column is pinned at the base, that all continuous columns in the building will yield at the same drift, buildings have constant interstory height, and that the continuous column size does not change over the height of the frame. Since equations for $M_{c,max}$ and $DCF$ were developed for buildings with five to twenty stories, 0.001 $\leq \alpha \leq 0.001$ and for roof ductilities between 2.0 and 6.0 they should not be used outside of this range.

Example of Frame Demand Assessment

A twelve-story CBF with a 3.5 m interstory height, $h$, and a bay width, $B$, of 6.0 m is to be designed. Based on preliminary sizing the fundamental period of the frame is 1.2 s. The same size members were used in each set of three stories and capacity design is used to ensure that brace yielding rather than column axial yielding was the predominant yield mode. Braces, in sets of three stories going up the building, have lateral story strengths of 400, 300, 300, and 200 kN, respectively. At the base shear of 400 kN, the yield roof displacement due to the code lateral force distribution is 500 mm. Twenty columns are associated with the CBF on one side of the structure. A similar frame and number of gravity columns are on the other side of the structure. The value of $\Sigma EI/h^3$ for the columns in the CBF as well as the gravity columns attributable to one CBF is $20 \times EI/h^3 = 20 \times 200 \times 10^3 \text{mm}^4/416 \times 10^6 \text{mm}^2/(3500 \text{mm})^3 = 38810 \text{kn/m}$ in the base story. The combined flexural strength of the continuous columns is 6090 kN/m. The steel braces have a yield strength of 350 MPa. The acceptable drift limit for the collapse performance state considered is assumed to be 2.5%.

1. Since the frame period is long, the equal displacement method is used to estimate the roof displacement of the inelastically responding structure, $\Delta_y$, as 500 mm.

2. The roof ductility, $\mu$, base story shear strength, $V_{f,base}$, story yield displacement ignoring column axial shortening effects (using $B = L_{brace} \cos \theta$ where $\theta$ is the brace angle from the horizontal), $\Delta_{1y}$, story stiffness, $k$, column stiffness ratio, $\alpha$, and the frame strength distribution, $\beta$, are given in the following.

$$\mu = \Delta_r/\Delta_y = 500 \text{ mm}/125 \text{ mm} = 4.0$$

$$V_{f,base} = 400 \text{ kN}$$
The following was shown.

3. The parameter \( \beta \), the column mechanism moment, \( M_{c,\text{mech}} \), the pushover moment at the maximum ductility, \( M_{c,\text{max}} \), and the drift concentration factor, \( DCF \), are

\[
\beta' = (1 - 1.3/12) 0.40^{1.25} + 0.5/12 = 0.273
\]

\[
M_{c,\text{mech}} = 0.192 (0.273) 400 \text{ kN} (3.5 \text{ m}) 12 = 881 \text{ kN m}
\]

\[
M_{c,\text{max}} = 881 \text{ kN m} (4.0 (3 - 0.4)/(3 + 0.4 12^2/5)/1.02)^{0.65}
\]

\[
= 716 \text{ kN m}
\]

\[
DCF_{\text{mech}} = \max \left\{ \frac{1 + 0.64 (4.0 - 1)^{0.5} (1 - 0.273/3)}{0.273^{0.310}} \frac{10^{0.273}}{0.120} \right\}^{1/0.273} = 1.42
\]

\[
= 1.69
\]

4. The maximum dynamic drift concentration factor, \( DCF_d \), is equal to DCF of 1.69 since \( \beta \) is greater than 0.3 and column moment, \( M_{c,\text{max}} \), considering dynamic effects is

\[
M_{c,\text{max}} = M_{c,\text{max}} + 0.000 108 (400 \text{ kN})
\]

\[
\times (42 \text{ m}) 4^{0.875} 1.0^{0.36} 12^{1.25}
\]

\[
= 716 \text{ kN m + 137 kN m = 853 kN m}
\]

5. The maximum story drift, \( \theta_{\text{max}} \), is 0.5 m/42 m (1.69) = 2.0%. which is less than the acceptable limit of 2.5%. Also, \( M_{c,\text{max,d}} < 6090 \text{ kN m} \) so yielding is not expected. The frame is therefore satisfactory.

Conclusions

Pushover and dynamic inelastic time history analyses were performed on concentrically braced frames with continuous columns to investigate the effect of these columns on structural demands. The following was shown.

1. Present design guidelines for concentrically braced and eccentrically braced frames have no explicit provisions to provide column flexural stiffness over the height of frames. However, if no, or very low column flexural stiffness exists, a soft story mechanism and large story drift demands will result from design level earthquake shaking.

2. Continuous columns over the height of the structure may be seismic or gravity columns. As the combined stiffness of these columns increases, story drift concentration is reduced. Since peak moments in the continuous columns may occur at more than one half of the way up the building height, it is important that column splices be sufficient to carry the expected moments.

3. Relationships were developed to estimate the drift concentration and column moment demand for two-story and multistory frames subject to pushover analysis.

4. Dynamic shaking effects caused the peak drift concentration and column moment demand to change from the static case. Empirical methods to assess these effects were proposed.

5. A procedure to estimate the likely drift concentration in frames of different heights was developed and an example was provided.

6. While this study has concentrated on the behavior of CBFs, concepts and methods described are also applicable to other frame types. This includes eccentrically braced frames as well as moment frames which may be designed either for predominantly beam yielding or for predominantly seismic column yielding.

Notation

The following symbols were used in this paper:

- \( B \) = braced bay width;
- \( DCF \) = drift concentration factor for static analysis;
- \( DCF_d \) = drift concentration factor for dynamic analysis;
- \( DCF_m \) = drift concentration factor for static analysis at formation of mechanism;
- \( DCF_{\text{B}=0.3} \) = drift concentration factor for static analysis assuming \( \beta = 0.3 \);
- \( DCF_{\text{B}=1} \) = drift concentration factor for static analysis assuming \( \beta = 1 \);
- \( d_c \) = displacement at center of column in two-story frame;
- \( d_i \) = interstory displacement for story \( i \);
- \( d_{iy} \) = story yield displacement at top of frame;
- \( d_1 \) = story 1 interstory displacement;
- \( d_{iy1} \) = story 1 interstory yield displacement;
- \( d_2 \) = story 2 interstory displacement;
- \( E \) = column flexural stiffness;
- \( F_i \) = point force at top of column;
- \( F_{\text{roof}} \) = point force at top of frame;
- \( F_{\text{base}} \) = force applied at level 1;
- \( F_{\text{2}} \) = force applied at level 2;
- \( G \) = shear stiffness of the frame;
- \( G_{\text{base}} \) = shear stiffness at the base of the frame;
- \( h \) = frame height;
- \( h_i \) = interstory height;
- \( h_{i,j} \) = interstory height of level \( i \);
- \( h_1 \) = interstory height of story 1;
- \( h_2 \) = interstory height of story 2;
- \( i \) = story counter;
- \( j \) = counter;
- \( k \) = story stiffness;
- \( k_i \) = lateral stiffness for story \( i \);
- \( k_1 \) = stiffness of story 1;
- \( k_2 \) = stiffness of story 2;
- \( L_b \) = brace length;
- \( M \) = moment;
- \( M_c \) = column moment;
- \( M_{cc,\text{base}} \) = moment at base of continuous column;
- \( M_{cc,\text{mech}} \) = maximum column moment in continuous frame at mechanism formation;
- \( M_{c,\text{max}} \) = maximum column moment;
\[ M_{c,\text{mech}} = \text{maximum column moment in discrete frame at mechanism formation}; \]
\[ M_{cm,\text{max}} = \text{maximum column moment in frame with discrete number of stories}; \]
\[ N = \text{number of stories}; \]
\[ P = \text{force at center of column}; \]
\[ R = \text{response modification factor}; \]
\[ S_{\text{ID}} = \text{IBC (ICCI, 2000) soil condition}; \]
\[ V = \text{shear force}; \]
\[ V_{c} = \text{column shear force}; \]
\[ V_{c1} = \text{column shear force in story 1}; \]
\[ V_{c2} = \text{column shear force in story 2}; \]
\[ V_{f} = \text{frame shear force}; \]
\[ V_{f,\text{base}} = \text{frame shear strength in story 1}; \]
\[ V_{fc} = \text{frame shear strength}; \]
\[ V_{f1} = \text{frame yield shear force}; \]
\[ V_{f1y} = \text{story yield force in story 1 of frame}; \]
\[ V_{f2} = \text{frame shear force in story 2}; \]
\[ V_{y} = \text{story shear for level i}; \]
\[ V_{\text{force, demand}} = \text{shear force demand if there is no continuous column}; \]
\[ V_{s,\text{base}} = \text{system base shear resistance}; \]
\[ V_{t1} = \text{shear force in story 1}; \]
\[ V_{t2} = \text{shear force in story 2}; \]
\[ w = \text{force per unit length}; \]
\[ w_{c} = \text{force per unit length on column}; \]
\[ w_{c1} = \text{force per unit length at top of column}; \]
\[ w_{f} = \text{force per unit length on frame}; \]
\[ w_{f1} = \text{force per unit length at top of frame}; \]
\[ w_{f1} = \text{force per unit length if there is no continuous column}; \]
\[ w_{f1} = \text{force per unit length at top of structure if there is no continuous column}; \]
\[ w_{f1} = \text{distance from base of structure}; \]
\[ x = \text{column stiffness ratio}; \]
\[ x = \text{frame strength distribution parameter}; \]
\[ \beta = \text{frame strength distribution parameter for frame with discrete number of stories}; \]
\[ \Delta_{1} = \text{roof displacement}; \]
\[ \Delta_{\text{mech}} = \text{roof displacement at mechanism formation}; \]
\[ \Delta_{f1} = \text{frame yield displacement at frame yield}; \]
\[ \Delta_{f1y} = \text{frame displacement at frame yield if there is no continuous column}; \]
\[ \Delta_{1} = \text{displacement at level 1}; \]
\[ \Delta_{1} = \text{yield displacement of first story}; \]
\[ \Delta_{2} = \text{displacement at level 2}; \]
\[ \Delta_{2y} = \text{displacement at level 2 at yield in 2 story frame}; \]
\[ \epsilon_{b} = \text{brace material yield strain}; \]
\[ \mu_{1} = \text{first story ductility}; \]
\[ \theta = \text{angle of brace to horizontal}; \]
\[ \theta_{c} = \text{column base rotation}; \]
\[ \theta_{c1m} = \text{column base rotation alone at mechanism formation}; \]
\[ \theta_{c1m} = \text{column top rotation alone at mechanism formation}; \]
\[ \theta_{f1m} = \text{peak frame drift at mechanism formation.} \]

**References**


